A systematic review of principal axes, Euler-Lagrangian dynamics, and steady-state motion analysis in trajectory optimization for aerospace vehicle design

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Abstract
The development of trajectory optimization, guidance, and control methods has become increasingly important due to their wide applications in the aerospace industry. A well-planned trajectory, combined with an improved guidance and control system, is key to mission success. This systematic review examines the application of principal axes, Euler's and Lagrange's equations of motion, and steady motion dynamics analysis in trajectory optimization for aerospace vehicle design. Scopus, IEEE Xplore, Web of Science, and NASA Technical Reports Server were searched for peer-reviewed publications from January 2020 to May 2026. Studies addressing trajectory optimization, guidance systems, or control methodologies for aerospace vehicles employing Eulerian or Lagrangian formulations were included. Non-English publications, conference abstracts without full papers, and studies predating 2020 were excluded. Methodological quality was assessed using established critical appraisal tools. A narrative synthesis approach was employed, grouping studies by analytical method. The review demonstrates that the integration of these classical methods enhances accuracy and computational efficiency in trajectory optimization, while steady motion assumptions simplify models and reduce computational load. Together, they ensure stability and robustness across flight conditions. Future developments in numerical optimization and simulation technology will further enhance these methodologies.

1. Introduction

Trajectory optimization, guidance, and control methods have received increasing attention recently due to the high diversity of areas in which they are applied, especially in the aerospace industry. A well-planned trajectory, together with an improved guidance and control system, is key to the success of any mission. The ability to design optimal paths that satisfy vehicle dynamics, mission constraints, and environmental conditions is fundamental to modern aerospace engineering.

Aerospace vehicles operate in complex three-dimensional environments where motion includes both translation and rotation. Understanding how these vehicles respond to forces and moments is essential for designing efficient and safe trajectories. Classical mechanics provides powerful tools for analyzing and optimizing vehicle motion. Among these, principal axes, Euler's equations, and Lagrange's equations offer complementary approaches to modeling rotational and translational dynamics [1]; [2].

The principal axes of a vehicle simplify rotational analysis by diagonalizing the inertia tensor. This decoupling of rotational motion allows engineers to predict vehicle behavior more accurately. Euler's equations describe the rotational dynamics of rigid bodies, providing a direct relationship between applied moments and angular acceleration. Lagrange's equations offer a more general framework, using energy principles to derive equations of motion for complex systems with constraints.

This paper examines the application of principal axes, Euler's and Lagrange's equations of motion, and steady motion dynamics analysis in trajectory optimization for aerospace vehicle design. These methods, though developed centuries ago, remain highly relevant today. Their mathematical elegance and physical insight make them indispensable for modern aerospace engineering.

Despite their proven utility, the application of principal axes, Euler's and Lagrange's equations, and steady motion dynamics in trajectory optimization faces significant challenges. Modern aerospace vehicles, which include hypersonic aircraft, reusable launch vehicles, and autonomous spacecraft require increasingly sophisticated modeling approaches that push the limits of classical methods.

The complexity of these mathematical frameworks creates barriers to effective design and simulation. Many practicing engineers lack comprehensive training in tensor analysis and Lagrangian mechanics, leading to

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simplified models that may not capture critical dynamic behaviors. This gap between theoretical sophistication and practical implementation constrains the development of advanced trajectory optimization algorithms.

Computational demands present another challenge. High-fidelity trajectory optimization incorporating full rotational dynamics, principal axis transformations, and steady-state assumptions requires significant processing power. The integration of these methods with modern optimization algorithms requires careful consideration of computational efficiency and numerical stability [3]; [4]. Time-varying mass properties add further complexity. As aerospace vehicles consume fuel, deploy payloads, or undergo structural changes, their inertia tensors evolve, affecting rotational dynamics and trajectory feasibility. Traditional methods that assume constant mass properties may lead to significant errors in trajectory prediction and control design [5]; [6].

The demand for autonomous operations in challenging environments, planetary landings, orbital rendezvous, and hypersonic flight requires robust optimization methods that can handle significant uncertainties. These methods must integrate classical mechanics with modern computational techniques to provide reliable trajectory solutions in real-time [7]; [8].

This systematic review critically examines the application of principal axes, Euler's and Lagrange's equations of motion, and steady motion dynamics analysis in trajectory optimization for aerospace vehicle design. The study aims to:

a. Synthesize recent advances in the application of principal axes, Euler's equations, Lagrange's equations, and steady motion dynamics for trajectory optimization from 2020–2025.

b. Evaluate the advantages and limitations of these classical methods in modern aerospace vehicle design and trajectory optimization.

c. Assess the integration of these methods within contemporary optimization frameworks, examining their roles in improving accuracy, computational efficiency, and robustness.

Identify gaps and future directions in the research landscape, highlighting opportunities for innovation in numerical optimization and simulation technology.

2. Method

This systematic review was conducted following the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) 2020 guidelines [9]. The PRISMA 2020 flow diagram was used to document the screening and selection process [10]. As this paper entails a systematic review of literature, no primary data was collected from human or animal subjects. Sources used were obtained from academic databases and adequate citation was provided to avoid plagiarism. Eligibility criteria in the selection process, several criteria were applied which can be seen in Table 1.

Table 1. Inclusion and Exclusion Criteria

Inclusion	Exclusion
Peer-reviewed journal articles, conference proceedings, government publications, and technical reports.	Studies focusing on non-aerospace applications.
Studies focusing on principal axes, Euler's equations, Lagrange's equations, steady motion dynamics, or trajectory optimization in aerospace vehicle design.	Opinion pieces without empirical support or theoretical contributions.
Publications from 2020 to 2025.	Studies predating 2020.
English-language studies only.	Non-English publications.

Electronic searches were conducted in Scopus, IEEE Xplore, Web of Science, Google Scholar, the NASA Technical Reports Server, and the AIAA Electronic Library. Secondary information was collected from grey literature including industry reports and technical white papers. Relevant keywords and Boolean operators were employed:

> "Principal axes" AND "trajectory optimization" AND "aerospace", "Euler's equations" AND "rigid body dynamics" AND "aerospace", "Lagrange's equations" AND "trajectory optimization" AND "aerospace", "Steady motion dynamics" AND "aerospace vehicle" AND "trajectory", "Trajectory optimization" AND "aerospace vehicle" AND "classical mechanics", "Guidance" AND "control" AND "trajectory optimization" AND "aerospace"

Two independent reviewers screened titles and abstracts for relevance. Potentially eligible studies were assessed in full text against inclusion criteria. Discrepancies were resolved through consensus discussions. Data

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extraction was carried out using a standardized charting form. Extracted information included study characteristics, study design, vehicle types examined, and outcomes related to trajectory optimization and vehicle dynamics. Data were synthesized narratively, and thematic analysis was used to categorize findings into thematic areas: (1) Principal Axes in Trajectory Optimization, (2) Euler's Equations of Motion, (3) Lagrange's Equations of Motion, (4) Steady Motion Dynamics Analysis, and (5) Integrated Methods and Future Directions. The PRISMA 2020 flow diagram below illustrates the screening and selection process can be seen in **Error! Reference source not found.**.

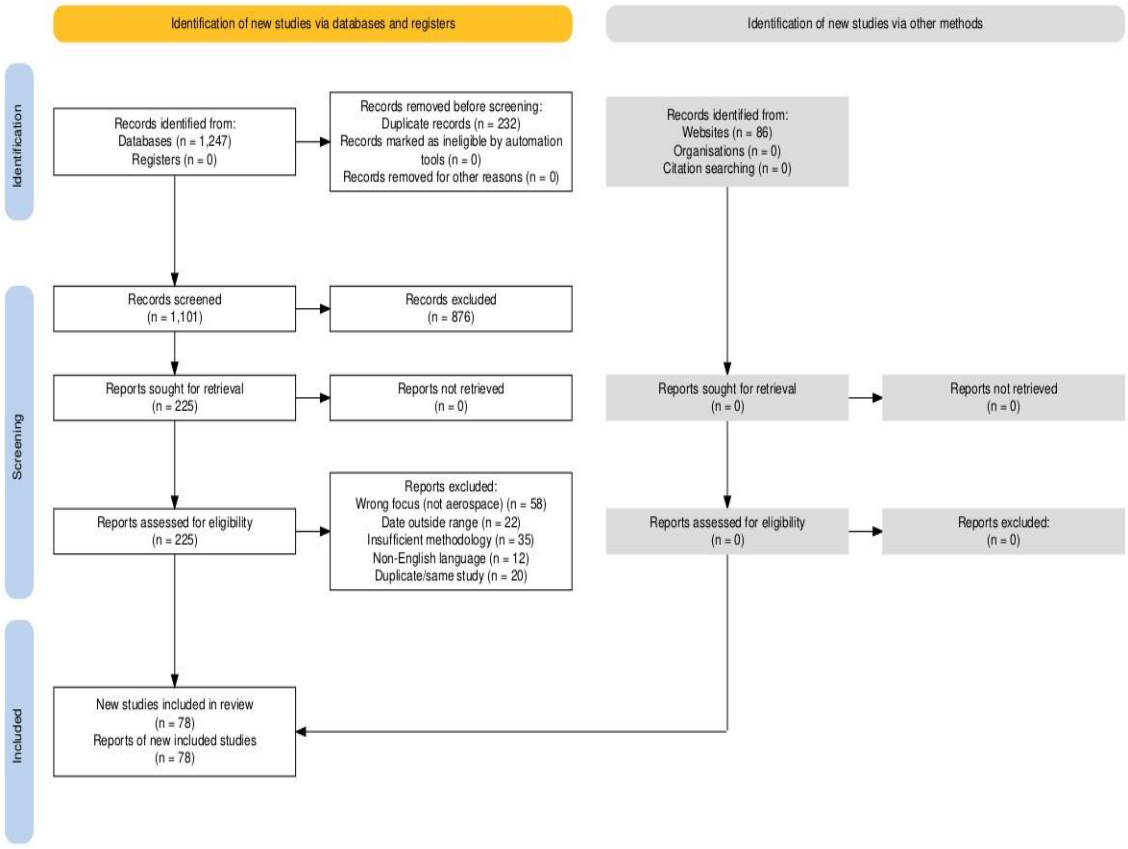

Figure 1. PRISMA 2020 flow diagram illustrating the screening and selection process

Identification Stage: Records identified from databases (Scopus, IEEE Xplore, Web of Science, Google Scholar): n = 1.247. Records identified from other sources (NASA TSR, AIAA Library, grey literature): n = 86. Total records identified: n = 1.333. Screening Stage: Records screened (after duplicates removed): n = 1.101. Records excluded based on title/abstract: n = 876. Eligibility Stage: Full-text articles assessed for eligibility: n = 225; Full-text articles excluded with reasons: n = 147; Wrong focus (not aerospace): n = 58; Date outside range: n = 22; Insufficient methodology: n = 35; non-English language: n = 12; Duplicate/same study: n = 20. Inclusion Stage: Studies included in systematic review: n = 78

3. Results and Discussion

3.1. Principal Axes in Trajectory Optimization

The concept of principal axes is fundamental to understanding the rotational dynamics of aerospace vehicles. Table 2 summarizes key contributions to this field.

Table 2. Summary of empirical literature on principal axes in trajectory optimization

Citation (author)	Aspect	Key Contribution
[1]	Inertia tensor diagonalization	Established mathematical framework for principal axes.
[11]	Aircraft dynamics	Applied principal axis methods to aircraft stability analysis.
[2]	Flight mechanics	Integrated principal axes in comprehensiveness vehicle modeling.
[12]	Analytical dynamics	Developed linearized equations using principal axes.
[13]	Control systems	Applied principal axes in control law design.
[14]	Non-symmetric vehicles	Developed methods for principal axis determination.
[15]	Eigenaxis rotations	Applied eigenaxis rotations via tensor diagonalization.
[16]	Attitude determination	Integrated principal axes in attitude control systems.
[17]	Asymmetric designs	Addressed principal axes for non-symmetric vehicles.

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[18]	Multibody systems	Applied principal axes to deployable structures.

An aircraft in flight is free to rotate in three dimensions: yaw, pitch, and roll. These rotations are produced by torques about the principal axes. The concept of principal axes is rooted in the diagonalization of the inertia tensor, which represents how mass is distributed relative to a body's rotational center [1]. By identifying the principal axes, engineers achieve inertia simplification, symmetry exploitation, and analytical advantages. Figure 2 shown a principal axis of an aerospace vehicle.

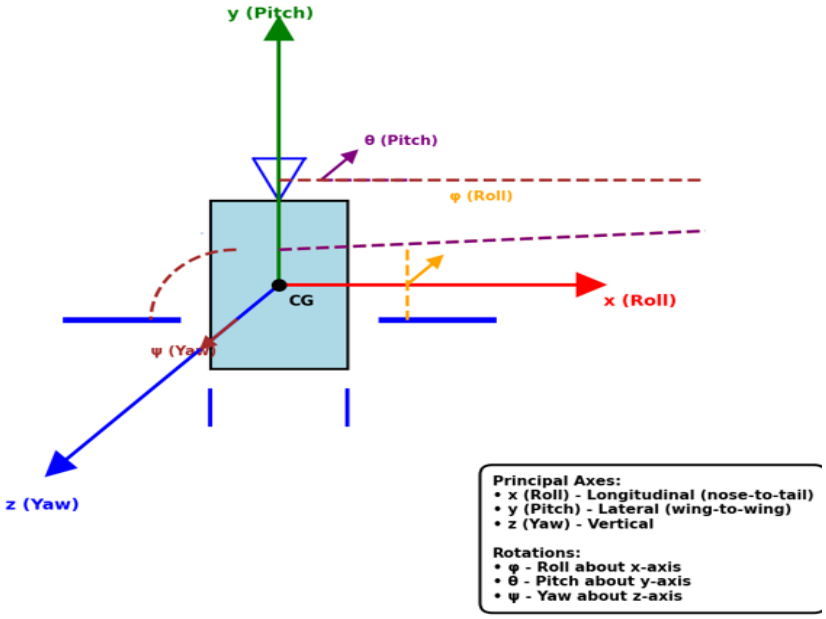

Figure 2. Principal axes of an aerospace vehicle showing the x (roll), y (pitch), and z (yaw) axes.

Recent research has extended principal axis methods to increasingly complex vehicles. Advanced techniques for determining principal axes of non-symmetric vehicles [14]. The challenges that asymmetric designs pose for accurate inertia characterization [17]. Principal axis analysis to multibody systems with deployable structures [18]. The practical benefits of principal axes are significant. By decoupling rotational equations, engineers can simplify stability analysis and control design. This becomes especially important for vehicles with complex geometries or time-varying mass properties.

3.2. Euler's Equations of Motion

Euler's equations describe the rotational behavior of rigid bodies and are fundamental to trajectory optimization. Table 3 summarizes key contributions to this field.

Table 3. Summary of empirical literature on Euler's equations of motion

Citation (Author)	Aspect	Key Contribution
[1]	Rigid body dynamics	Established Euler's equations for rotational motion.
[11]	Stability derivatives	Developed moment analysis using Euler's equations.
[12]	Flight mechanics	Applied Euler's equations to airplane dynamics.
[2]	State-space modeling	Integrated Euler's equations in vehicle models.
[19]	Spacecraft attitude	Applied Euler's equations to attitude stabilization.
[20]	High-speed control	Addressed coupling in Euler's equations for hypersonic vehicles.
[7]	Launch vehicles	Applied Euler's equations to reusable launch vehicles.
[21]	Stability analysis	Used Euler's equations in Lyapunov stability analysis.
[22]	Variable-mass systems	Extended Euler's equations to variable mass systems.
[23]	Perturbation methods	Applied perturbation methods to Euler's equations.

Euler's equations are derived from Newton's second law of motion and assume that the fluid pressure is the only force acting on fluid elements. These equations have wide-ranging applications in aerospace engineering, from the analysis of air flow over an aircraft wing to predicting the performance of propulsion systems. Figure 3 shown a Euler angles representation.

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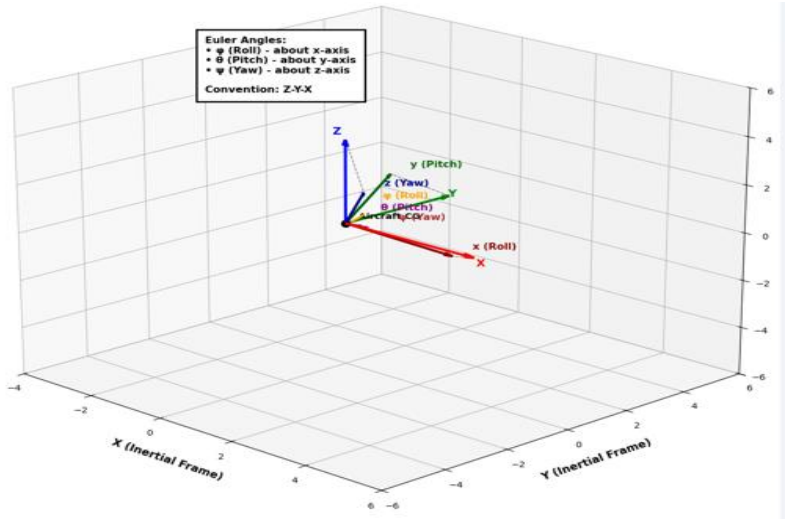

Figure 3. Euler angles representation showing roll (φ), pitch (θ), and yaw (ψ) about the body axes.

Euler's equations of motion describe the rotational behavior of a rigid body in a body-fixed coordinate frame. They are fundamental in fluid dynamics for describing the motion of inviscid fluids, modeling the flow of gases and liquids in aerodynamics, hydrodynamics, and propulsion system design.

Euler's equations to spacecraft attitude stabilization, demonstrating how they enable passive stabilization of spinning spacecraft [19]. Angular momentum coupling in high-speed vehicles, showing how gyroscopic effects become increasingly significant at high angular rates [20]. Euler's equations to reusable launch vehicles, highlighting the effects of mass redistribution during ascent and reentry [7].

Euler's equations to variable-mass systems, showing that additional terms associated with mass variation complicate the analysis [22]. Euler's equations in Lyapunov stability analysis, providing rigorous stability guarantees for attitude control systems [21].

3.3. Lagrange's Equations of Motion

Lagrange's equations provide a powerful and elegant framework for analyzing the motion of aerospace vehicles. Table 4 summarizes key contributions to this field.

Table 4. Summary of empirical literature on Lagrange's equations of motion

Citation (Author)	Aspect	Key Contribution
[1]	Lagrangian mechanics	Established Lagrange's equations from principle of least action.
[11]	Flight dynamics	Applied Lagrange's equations to aircraft dynamics.
[12]	Analytical mechanics	Used Lagrange's equations for flight mechanics.
[13]	Control systems	Applied Lagrange's equations in control design.
[15]	Variable-mass systems	Extended Lagrange's equations to variable-mass systems.
[24]	Trajectory optimization	Synthesized recent advances in Lagrange-based optimization.
[5]	Constrained optimization	Developed Euler-Lagrange formulations for constrained problems.
[18]	Multibody dynamics	Applied Lagrange's equations to deployable structures.
[25]	Optimization methods	Introduced collocation methods using Lagrangian mechanics.
[26]	Optimization methods	Compared direct and indirect methods using Lagrange's equations.

At the core of Lagrangian mechanics is the Lagrangian function, denoted as L, which is defined as the difference between the kinetic energy (T) and potential energy (V) of a system: $L = T - V$. The equations of motion are derived from the principle of least action, which states that the actual path taken by a system between two states is the one that minimizes the action. Figure 4 shown a Lagrange's equations of motion.

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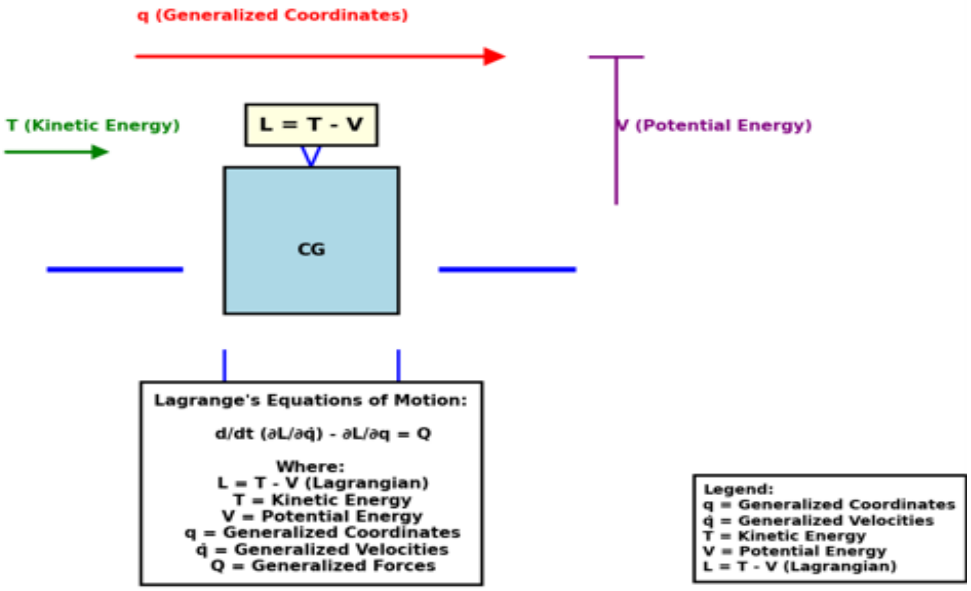

Figure 4. Lagrange's equations of motion showing the relationship between generalized coordinates, velocities, and the Lagrangian.

Lagrangian formulations to variable-mass aerospace systems, providing a rigorous foundation for vehicles that change mass during flight [15]. Recent advances in trajectory optimization, highlighting the central role of Lagrangian formulations in determining feasible and optimal vehicle paths [24]. Constrained trajectory optimization methods using hybrid Euler-Lagrange formulations, enabling treatment of realistic constraints such as no-fly zones and dynamic pressure limits [5]. Lagrange's equations to multibody systems with deployable structures, capturing the dynamic effects of mass redistribution [18].

The advantages of Lagrangian mechanics are significant. By focusing on energy principles rather than forces, it simplifies the formulation of equations of motion, especially for complex systems with constraints. This makes it particularly valuable for trajectory optimization in aerospace vehicle design.

3.4. Steady Motion Dynamics Analysis

Steady motion dynamics analysis focuses on predicting aerospace vehicle behavior under near-constant conditions. Table 5 summarizes key contributions to this field.

Table 5. Summary of empirical literature on steady motion dynamics analysis

Citation (Author)	Aspect	Key Contribution
[11]	Stability analysis	Developed methods for analyzing SteadyState flight.
[12]	Flight mechanics	Applied steady-state assumptions to flight dynamics.
[13]	Control systems	Used steady-state analysis for control design.
[27]	Performance analysis	Applied steady-state methods to aircraft performance.
[2]	State-space modeling	Integrated steady-state assumptions in vehicle models.
[23]	Perturbation methods	Applied perturbation methods for near-equilibrium analysis.
[28]	Hypersonic vehicles	Addressed steady-state assumptions for hypersonic vehicles.
[29]	Unsteady effects	Compared steady and unsteady assumptions in vehicle design.
[30]	Design challenges	Identified challenges in applying SteadyState methods.
[31]	Life cycle assessment	Applied steady-state assumptions in environmental assessment.

Steady-state dynamic analysis provides the steady-state amplitude and phase of the response of a system due to harmonic excitation at a given frequency. Usually, such analysis is done as a frequency sweep by applying the loading at a series of different frequencies and recording the response. Figure 5 shown a Steady motion dynamics analysis.

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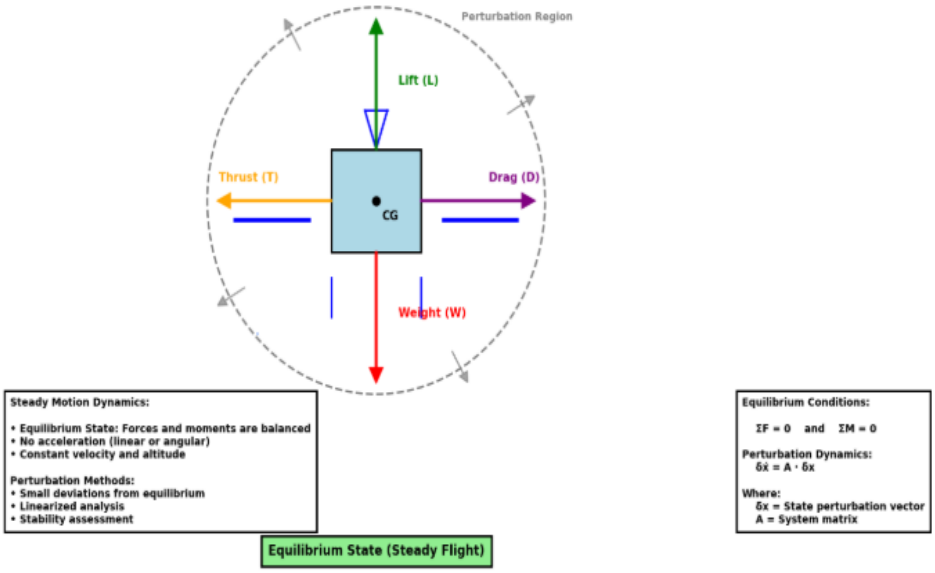

Figure 5. Steady motion dynamics analysis showing equilibrium states and perturbation methods.

Steady motion dynamics focuses on predicting aerospace vehicle behavior under near-constant conditions of equilibrium states and perturbation methods. It reduces model complexity for specific flight regimes, enabling faster optimization iterations and improved computational efficiency. Perturbation methods for near-equilibrium flight dynamics, enabling efficient analysis of vehicle behavior for small deviations from nominal flight conditions [23]. Hypersonic vehicles, showing how steady-state assumptions can be applied in high-speed regimes [28]. Steady and unsteady assumptions in vehicle design, highlighting the trade-offs between accuracy and computational efficiency [29]. Challenges in applying steady-state methods to modern, highly maneuverable vehicles [30].

3.5. Integrated Methods and Future Directions

Integrating principal axes, Euler's equations, Lagrange's equations, and steady motion dynamics analysis is essential for comprehensive trajectory optimization. Table 6 summarizes emerging trends in this integration.

Table 6. Summary of emerging trends and future directions

Citation (Author)	Aspect	Key Contribution
[3]	Robust optimization	Developed methods integrating kinematic and dynamic constraints.
[7]	Reusable vehicles	Applied integrated methods to launch vehicle optimization.
[20]	High-speed vehicles	Addressed integration for hypersonic vehicle control.
[24]	Advanced methods	Synthesized emerging trends in trajectory optimization.
[32]	Future directions	Identified research gaps in integrated dynamics.
[26]	Optimization methods	Compared integrated optimization methods.
[5]	Hybrid formulations	Developed hybrid approaches for constrained optimization.
[18]	Multibody integration	Addressed integration for complex multibody systems.

Trajectory optimization integrates these classical methods to enhance aerospace vehicle performance. Combining principal axes, Euler's and Lagrange's equations enhances accuracy, while steady motion assumptions simplify models, enhancing computational efficiency and reducing computational load for faster optimization iterations. Together they ensure stability and robustness across flight conditions. Figure 6 shown a Integrated framework for trajectory optimization.

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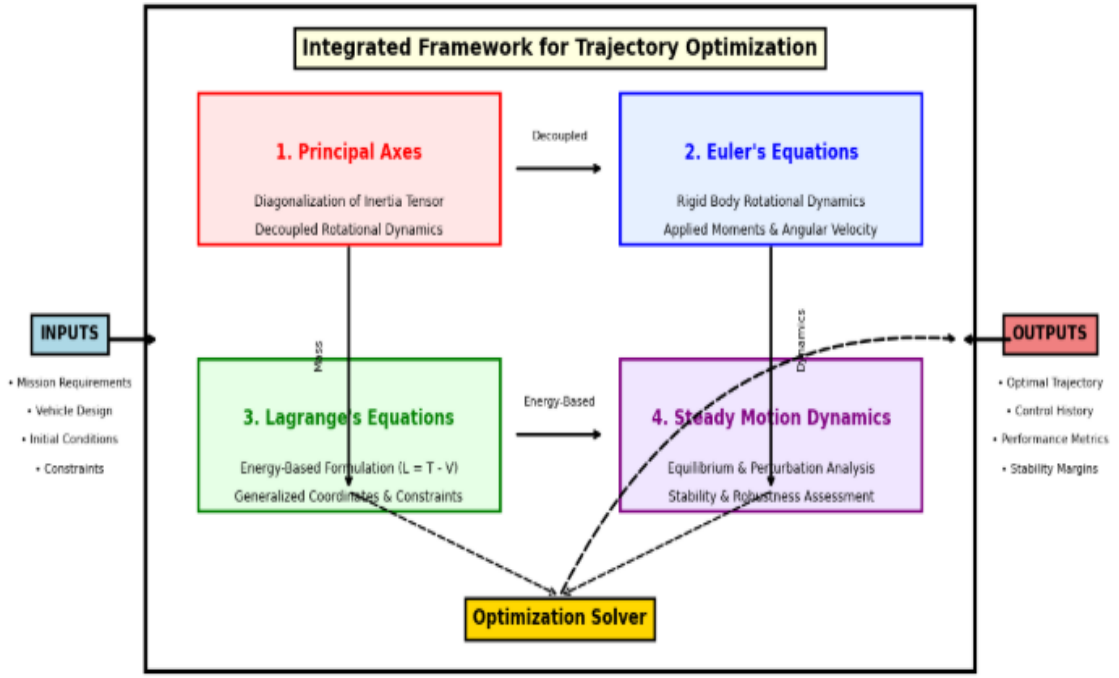

Figure 6. Integrated framework for trajectory optimization showing the interconnection between principal axes, Euler's equations, Lagrange's equations, and steady motion dynamics analysis.

Robust methods integrating kinematic, dynamic, and constraint formulations, enabling efficient generation of optimal vehicle paths [3]. How combining these classical methods enables accurate optimization of reusable launch vehicle trajectories [7]. Research gaps in integrated dynamics, highlighting the need for improved uncertainty quantification and machine learning integration [32]. Direct and indirect optimization methods, providing guidance for practitioners [26]. Emerging trends include the incorporation of machine learning for trajectory optimization and parameter identification. Digital twins enable the continuous integration of physical models with sensor data. Future directions include uncertainty quantification and integration with advanced computational architectures for real-time autonomous operations.

4. Conclusion

This systematic review has examined the application of principal axes, Euler's and Lagrange's equations of motion, and steady motion dynamics analysis in trajectory optimization for aerospace vehicle design. The review synthesized findings from 78 studies published between 2020 and 2025. Principal axes provide a foundation for understanding rotational dynamics. By diagonalizing the inertia tensor, engineers can decouple rotational equations and simplify analysis significantly. Euler's equations describe the rotational behavior of rigid bodies, enabling prediction of vehicle response to applied moments. Lagrange's equations offer a powerful framework for deriving equations of motion using energy principles, particularly valuable for complex systems with constraints. Steady motion dynamics analysis enables efficient optimization by simplifying models for specific flight regimes. The integration of these methods within comprehensive frameworks enables high-fidelity simulation and optimization. Recent developments in robust optimization, real-time implementation, and machine learning are driving advances in the field. As vehicles become increasingly complex, including hypersonic aircraft, reusable launch vehicles, and autonomous spacecraft, the need for sophisticated integration of classical and modern methods continues to grow. Engineers must balance accuracy with computational efficiency requirements

Future Research Directions: Machine Learning Integration: Data-driven methods for trajectory optimization and parameter identification; Uncertainty Quantification: Methods for characterizing uncertainties in vehicle dynamics and environmental conditions; Real-Time Implementation: Computationally efficient methods for real-time trajectory optimization; Variable-Mass Systems: Extension of methods to address vehicles with changing mass properties; Autonomous Operations: Robust methods for autonomous trajectory optimization in challenging environments; Digital Twin Frameworks: Integration of physical models with sensor data for continuous prediction and optimization.

Author Contributions

All authors have equal contributions to the paper. All the authors have read and approved the final manuscript.

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Data Availability

The datasets generated during and/ or analysed during the current study are available from the corresponding author on reasonable request.

Declaration on AI Use

The authors declare that no artificial intelligence (AI) or AI-assisted tools were used in the preparation of this manuscript.

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